

María and Her Trading Cards																			
Problem wording	<p>María holds an envelope containing trading cards in her left hand and three cards in her right.</p> <p><b>Identification of specific cases, recognition of structure and formulation of a conjecture</b></p> <p>1. To ensure students understand the word problem they should be asked them to represent it. For instance:</p> <p><i>Describe the number of cards María holds in words or in a drawing.</i></p> <p>Pooling group knowledge: each student presents and explains their representation.</p> <p>Sample questions:</p> <ul style="list-style-type: none"> <li>- <i>What do you think of (name)'s answer?</i></li> <li>- <i>How did you find your answer?</i></li> <li>- <i>Would you add anything to or delete anything from (name)'s answer?</i></li> </ul> <p>2. Working individually</p> <p>Each student performs the following task. They are asked to relate the number of cards in the envelope to the total held by María. They are instructed to indicate which matches are correct by drawing a line between the respective values.</p> <table style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> <tr> <td style="text-align: center;"><b>4</b></td> <td style="text-align: center;"><b>11</b></td> </tr> <tr> <td style="text-align: center;"><b>7</b></td> <td style="text-align: center;"><b>5</b></td> </tr> <tr> <td style="text-align: center;"><b>13</b></td> <td style="text-align: center;"><b>7</b></td> </tr> <tr> <td style="text-align: center;"><b>15</b></td> <td style="text-align: center;"><b>18</b></td> </tr> <tr> <td style="text-align: center;"><b>8</b></td> <td style="text-align: center;"><b>4</b></td> </tr> <tr> <td style="text-align: center;"><b>2</b></td> <td style="text-align: center;"><b>12</b></td> </tr> <tr> <td style="text-align: center;"><b>9</b></td> <td style="text-align: center;"><b>3</b></td> </tr> <tr> <td style="text-align: center;"><b>6</b></td> <td style="text-align: center;"><b>10</b></td> </tr> </tbody> </table> <p>3. Working collectively</p> <p>With the aid of their teacher, the students share their answers and arrange the information in a table. The table should be large enough to pin to the board so it can be filled in by the students. The columns are unnamed.</p>			<b>4</b>	<b>11</b>	<b>7</b>	<b>5</b>	<b>13</b>	<b>7</b>	<b>15</b>	<b>18</b>	<b>8</b>	<b>4</b>	<b>2</b>	<b>12</b>	<b>9</b>	<b>3</b>	<b>6</b>	<b>10</b>
																			
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**Validating a conjecture**

4. Working collectively

They should be introduced to the examples/cases listed below and asked to determine whether they are right or wrong (if wrong, to explain why).

Number of cards in the envelope	Number of cards of María
500	500
1.000	$1.000 + 3$
201	$201 + 1 + 1 + 1$
$150 + 3$	150
432	$432 + 2 + 1$

Other possible examples:

$14 - 3$	14
$765 - 3$	765
120	$120 - 3$

5. They can also be asked to fill in the blanks in a table indicating either the number of cards or the total held by María. A few examples can be taken from the table below.

The envelope contains	If María has...
13	$5 + 1 + 1 + 1$
20	$11 + 3$
345	$564 + 3$
	$1.000.000 + 2 + 1$

**Generalising a conjecture**

- If there are lots of cards in the envelope, how many does María have altogether?
- And if we never know how many cards there are in the envelope, how can we represent that unknown quantity?
- If there are  $Z$  cards in the envelope, how can we know how many María has altogether?
- Can you think of a way to write the number of cards if we don't know what that number is?
- If there are  $Z$  cards in the envelope, does María have  $Z$  altogether?
- If there are  $Z$  cards in the envelope, does María have  $Z+3$  altogether?
- If there are  $Z$  cards in the envelope, does María have  $Z+1+1+1$  altogether?
- If there are  $Z$  cards in the envelope, does María have  $Z+2+1$  altogether?

**Optional**

	<p>- If María has <math>Z</math> cards altogether, how can we know how many there are in the envelope?</p> <p>- If there are <math>W-3</math> cards in the envelope, how many does María have altogether?</p>
Purpose	<ul style="list-style-type: none"> <li>• To apply a rule governing the function to specific numerical cases.</li> <li>• To generalise the functional relationship.</li> <li>• To generalise the functional relationship in cases involving an indeterminate quantity.</li> </ul>
Suggestions for classroom delivery	<p>Task 1 is meant to help students understand the word problem and prompt them to jot down the relationship '<math>x+3</math>'. If they propose numerical examples, ask:</p> <ul style="list-style-type: none"> <li>- What made you think of that number of cards?</li> <li>- Could there be some other number of cards?</li> <li>- Why are you sure the total number of cards is correct?</li> </ul> <p>The questions suggested for task 3 include:</p> <ul style="list-style-type: none"> <li>- How could we name the columns? If the first column is named 'number of cards in the envelope', what would the second one be named?</li> <li>- If there are ... cards in the envelope, how many does María have altogether? Why?</li> <li>- Which examples have no match? What would the match be?</li> <li>- If we don't know how many cards there are in the envelope, how can we know if our answers are right or wrong?</li> </ul> <p>Note: If the students explain orally how they found the answer, their explanations should be written it in alongside the table, as a third column. They may also be asked to read the table from left to right, so they see the inverse relationship (optional - inverse relationship).</p> <p>In the examples in tasks 4 and 5, expressed as additions, students are told not to operate to help them develop a structural vision. They may be asked: If you don't add or subtract, can you know how many cards there are in the envelope or how many María holds?</p> <p>The examples involving letters are related to the numerical examples proposed in tasks 4 and 5. The idea is, if they identified the examples as correct in the earlier task, how do they handle them now?</p>