Tables and Gifts

Problem wording

Isabel is getting ready for her birthday party. She puts boxes containing little boxes for her guests on each table. She arranges a few tables in a row and sets one box at each end, as in the drawing.

Identification of specific cases and recognition of structure

In a classroom exercise, the teacher introduces the tasks and has students collectively fill in a table like the one below.

Number of tables	

Data to be included in the table

Direct relationship: students know the number of tables but not the number of boxes. The teacher says: ‘We’re going to organise the information given in the table’.

a) What words could we write in to head the second column?

b) How can we fill in the table using the information in the drawing?

c) How many boxes do we need for eight tables? How did you find the answer? (Note: show a drawing with eight tables and write the number 8 in the table. Tell students they can use drawings to find the answer or do the arithmetic in writing or mentally.)

c) How many boxes do we need for two tables? How did you find the answer?

c) How many boxes do we need for five tables? How did you find the answer?

Inverse relationship: students know the number of boxes but do not know the number of tables.

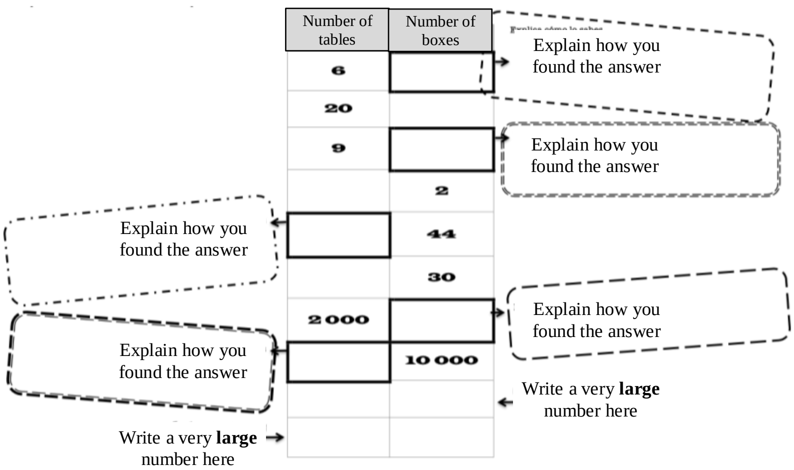
a) If Isabel uses 14 boxes, how can we show that on the table? c) How many tables does she need? How did you find the answer?

c) If she uses eight boxes, how many tables does she need? How did you find the answer?

c) If she uses 12 boxes, how many tables does she need? How did you find the answer?

Formulating and validating a conjecture

After the group has worked together on a number of specific examples or cases, students are given time to explore others and formulate and validate their conjectures, either individually or in small groups.



Generalising a conjecture

1. How did you find the answer to "how many tables there are" when you know the number of boxes?

2. How did you find the answer to "how many boxes there are" when you know the number of tables?

Other ways to pose the question

What statements are true (T) or false (F)? Correct the ones you think are false by changing them as necessary.

Number of tables	Number of Number of				Explanation
2	2	⇨	V	F	
4	8	⇨	V	F	
4 x 2	4	⇨	V	F	
13 - 2	13	⇨	V	F	
1.000	Two times 1000	⇨	V	F	
22	22 x 2	⇨	V	F	
5	5 + 5	⇨	V	F	
Half of 2 million	2 million	⇨	V	F	
10 : 2	10	⇨	V	F	

Explain in writing how many boxes you need for Q tables. [Indeterminate quantities can also be labelled differently or with another letter.]

What statements are true (T) or false (F)? Correct the ones you think are false by changing them as necessary.

a) There are twice as many tables as boxes.

b) When Isabel has 11 tables, she needs 21 boxes.

c) When Isabel has four tables, she needs 2x4 boxes.

d) When Isabel has 12 tables, she needs six boxes.

e) When Isabel has Z tables, she needs 2xZ boxes.

f) When Isabel has Z tables, she needs Q boxes.

g) When Isabel has Z tables, she needs Z+Z boxes.

h) When Isabel has Z tables, she needs Z boxes.

Purpose	• To build on specific cases to discover the rule governing the function.

	• To apply the rule governing the function to specific numerical cases. • To generalise the functional relationship. • To generalise the functional relationship in cases involving an indeterminate quantity.
Suggestions for classroom delivery	One way to help students recognise the variables is to ask them to complete the headings of the columns in the table proposed. They may suggest general terms such as ‘tables’ or ‘boxes’. In that case, tell them they need to be more specific by asking whether they refer to the number, the colour or the size of the boxes. Although that may sound overly obvious, it helps them identify the features to be represented. They are asked to explain only a few cases in the table task to make it more dynamic. The idea is for them to explain a few examples as fully and precisely as possible, i.e., mentioning the variables, the relationship between them or other elements of the context as necessary. In our experiences with children, we have observed that they sometimes claim to find the answers by ‘drawing’ or ‘counting’. While valid, such replies are imprecise. We therefore encourage them to explain orally or in writing what they drew or what they counted and how that related to the context. In the questions dealing with generalisation, teachers should stress that neither the number of tables nor the number of boxes is known. One way to get that idea across is to use keywords such as ‘a lot of tables’, ‘an unknown number of tables’ or ‘any number of tables’. Depending on their answers, students can be challenged by changing the initial conditions. They might be asked, for instance, how they would need to change the way to determine the number of boxes and tables if the birthday girl sits at one of the corners, as in the drawing.  The true/false questions include arithmetic expressions. All had been used by the students in the introductory classroom session to describe the idea ‘double’. Where students express that idea otherwise, their proposals should be reworded and included as true/false statements to prompt them to think about the possibility of describing the functional relationship in more than one way. By analysing students’ interpretation of arithmetic expressions, the teacher can determine whether their understanding is operational or structural. The second option favours algebraic thinking.